Coupling-Independent Capacitive Wireless Power Transfer with One Transmitter and Two Receivers using Frequency Bifurcation

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***Abstract*—Capacitive wireless power transfer utilizes capacitive coupling to transfer electrical energy wirelessly. However, with variation in distance or alignment, this coupling varies and as a result the efficiency and power transfer varies. In this work, we propose a frequency agile mode, using frequency bifurcation, that allows for a nearly coupling-independent regime for a capacitive wireless power transfer system with one transmitter and two receivers. The conditions for bifurcation are described and analytical expressions for the power and transducer gains are determined. It is shown that, when operating at the secondary resonances, nearly constant efficiency and power transfer to the load can be achieved.**

***Index Terms*—capacitive wireless power transfer, frequency bifurcation, single input multiple outputs**

I. Introduction

Capacitive wireless power transfer (CPT) is a form of wireless power transfer (WPT) that utilizes capacitive coupling to transfer electrical energy between one or more transmitters to one or more receivers without the need for physical connections. CPT offers several advantages, including flexibility in design, high efficiency, long-range operation, multi-device charging, environmental compatibility, and scalability, making it a promising technology for wireless power transfer in various applications, such as charging of mobile devices, powering sensors in remote locations, and energizing implantable medical devices [1]–[4].

As mentioned above, one of the advantages of CPT is multi-device charging, which refers to the ability of CPT to simultaneously charge or power multiple devices from a single transmitter, commonly referred to as a single input multiple outputs (SIMO) configuration. This can be particularly beneficial in scenarios where multiple devices must be charged simultaneously, such as in smart homes, offices, or public charging stations.

An interesting phenomenon observed in CPT is frequency bifurcation [5]–[7], where multiple frequencies exist for which the input impedance is matched (i.e., the input reactance equals zero), and hence, multiple resonant frequencies exist. This phenomenon is also observed in inductive wireless power transfer (IPT) [8]. By applying frequency bifurcation, a (nearly) constant efficiency and power transfer can be realized, even at fluctuating coupling, by sweeping the frequency to realize a purely resistive input impedance. This has been shown for single input single output (SISO) systems for both IPT [8] and CPT [5], [6].

The conditions for bifurcation in SIMO systems have been derived in [7]. However, to the best of our knowledge, a coupling-independent solution for SIMO CPT has not been presented in analytic form, and the coupling-independent regime due to this bifurcation has not been explored for these systems yet.

In this study, we will investigate the application of a frequency agile approach utilizing the bifurcation phenomenon to achieve a CPT system with one transmitter and two receivers that remains unaffected by fluctuations of the coupling. The system and a mathematical notation using the admittance matrix are presented, and the input admittance is used to determine the conditions for bifurcation. Analytical expressions for the power and transducer gains are derived, and, using the bifurcation conditions, expressed in normalized circuit parameters. The results are visually presented, using two illustrative examples, one with equal and one with unequal receivers, showing a nearly constant efficiency and power regime with significantly increased output power compared to a fixed frequency approach.

II. Methods

A. Circuit description

We consider the equivalent circuit of a CPT system with a single transmitter and two receivers, as shown in Fig. 1. The supply of the transmitter is represented by a sinusoidal current source with peak value I_S and shunt conductance G_S, and the purely restive load of the receivers is given by conductances

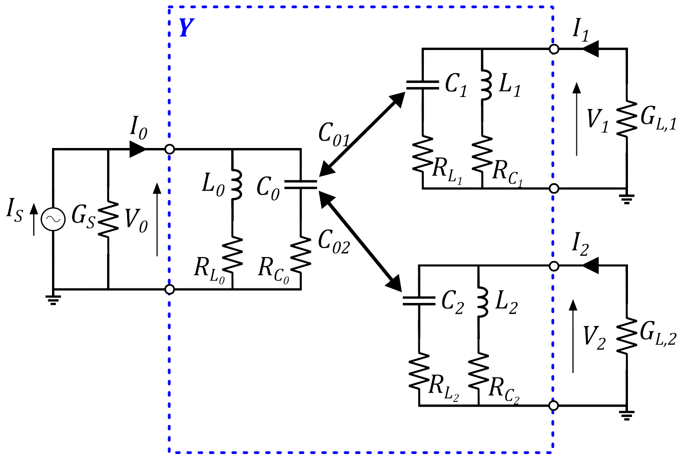

Fig. 1. Equivalent circuit representation of a single input, two outputs CPT system.

$G_{L,1}$ and $G_{L,2}$. The capacitive link is represented by the capacitances C_i $(i = 0, 1, 2)$, and their mutual coupling capacitances C_{01} and C_{02}. The shunt inductances L_i are used to create a resonant circuit by using an inductance of $1/\omega_0^2 C_i$, with ω_0 the operating angular frequency of the current source. The inductor and capacitor losses are taken into account by their equivalent series resistances (ESRs), R_{L_i} and R_{C_i}. Note that because of the introduction of these ESRs, the resonant frequency is slightly changed, however, this change is small for practical CPT systems and can therefore be neglected.

B. Admittance matrix

The CPT system can be considered as a multiport with one input port and two output ports, indicated in Fig. 1, which can be represented by an admittance matrix $\mathbf{Y}$ [9]:

$$\mathbf{Y} = \begin{bmatrix} y_{00} & y_{01} & y_{02} \\ y_{10} & y_{11} & y_{12} \\ y_{20} & y_{21} & y_{22} \end{bmatrix}. \tag{1}$$

The diagonal and off-diagonal terms of the admittance matrix $\mathbf{Y}$ are given by:

$$y_{xx} = \frac{1}{R_{L_x} + j\omega L_x} + \frac{1}{R_{C_x} + \frac{1}{j\omega C_x}}, \tag{2}$$

$$y_{xy} = -j\omega C_{xy}. \tag{3}$$

In order to generalize the circuit analysis, for any possible operating frequency and power levels, it is convenient to introduce normalized quantities. We define the normalized frequency u as:

$$u = \frac{\omega}{\omega_0}, \tag{4}$$

and the normalized ESRs and load conductances R_{C_i}, R_{L_i} and $g_{L,i}$ as:

$$r_{C_i} = \omega_0 C_i R_{C_i}, \tag{5}$$

$$r_{L_i} = \frac{R_{L_i}}{\omega_0 C_i}, \tag{6}$$

$$g_{L,i} = \frac{G_{L,i}}{\omega_0 C_i}. \tag{7}$$

For high-quality factors (i.e., efficient passive components), commonly encountered in practical CPT applications, we can simplify the admittance matrix with the normalized quantities by neglecting the squared normalized resistances with respect to u^2, and find the diagonal and off-diagonal entries of the approximate admittance matrix $\mathbf{Y_a}$ as [6]:

$$y_{xx} = \omega_0 C_x \left(r_{C_x} u^2 + \frac{r_{L_x}}{u^2} + j\frac{u^2 - 1}{u} \right), \tag{8}$$

$$y_{xy} = -j\sqrt{C_x C_y}\,\omega_0 k_{xy} u, \tag{9}$$

with k_{xy} the coupling coefficient, given by:

$$k_{xy} = \frac{C_{xy}}{\sqrt{C_x C_y}}. \tag{10}$$

C. Input admittance

The input admittance Y_{in} of the considered multiport system is given by [10]:

$$Y_{in} = y_{00} + \frac{y_{02} M_{02} - y_{01} M_{01}}{M_{00}}, \tag{11}$$

with M_{0i} the minor of the matrix $\mathbf{Y_a} + \mathbf{Y_L}$ and $\mathbf{Y_L}$ the diagonal load admittance matrix.

To simplify the notation, the following definitions are introduced:

$$a_i = 1 + 2g_{L,i} r_{C_i} \tag{12}$$

$$b_i = 2 - g_{L,i}^2 \tag{13}$$

$$c_i = 1 + 2g_{L,i} r_{L_i} \tag{14}$$

$$d = r_{L_0} + r_{L_1} + r_{L_2} \tag{15}$$

$$e = r_{C_0} + r_{C_1} + r_{C_2} \tag{16}$$

$$r_i = r_i = r_{L_i} + r_{C_i}. \tag{17}$$

We can express the real (g_{in}) and imaginary (b_{in}) parts of the normalized input admittance $y_{in} = Y_{in}/\omega_0 C_0$ by neglecting $r_{C_i}^2$, $r_{L_i}^2$ and $r_{C_i} r_{L_i}$ with respect to u^2 as:

$$g_{in} = r_{C0} u^2 + \frac{r_{L0}}{u^2} + \frac{k_{01}^2 u^2 \left(r_{C1} u^4 + g_{L1} u^2 + r_{L1} \right)}{a_1 u^4 - b_1 u^2 + c_1} + \frac{k_{02}^2 u^2 \left(r_{C2} u^4 + g_{L2} u^2 + r_{L2} \right)}{a_2 u^4 - b_2 u^2 + c_2}, \tag{18}$$

$$b_{in} = \frac{u^2 - 1}{u} \left(1 - \frac{k_{01}^2 u^4}{a_1 u^4 - b_1 u^2 + c_1} - \frac{k_{02}^2 u^4}{a_2 u^4 - b_2 u^2 + c_2} \right). \tag{19}$$

From (19) it is evident that the input susceptance equals zero if the normalized frequency u is one, i.e., at the resonant frequency ω_0. We call this the primary resonant frequency.

Furthermore, it is apparent that other frequencies exist where the input susceptance equals zero. We can find these other frequencies by substituting $u^2 = x$, and solving the quartic equation:

$$\frac{k_{01}^2 x^2}{a_1 x^2 - b_1 x + c_1} + \frac{k_{02}^2 x^2}{a_2 x^2 - b_2 x + c_2} = 1. \tag{20}$$

For a system with identical receivers (i.e., $a_1 = a_2 = a$, $b_1 = b_2 = b$ and $c_1 = c_2 = c$), this quartic equation reduces to:

$$\left(a - k_{01}^2 - k_{02}^2\right) x^2 - bx + c = 0. \tag{21}$$

For a CPT system that fulfills the requirement of a non-negative discriminant (i.e. $k_{01}^2 + k_{02}^2 < a - \frac{b^2}{4c}$) and $k_{01}^2 + k_{02}^2 < a$, only the primary resonant frequency is present for $b < 0$, and two distinct secondary frequencies exists for $b > 0$, following [7]:

$$u_{\pm} = \sqrt{\frac{b \pm \sqrt{b^2 - 4c\left(a - k_{01}^2 - k_{02}^2\right)}}{2\left(a - k_{01}^2 - k_{02}^2\right)}}. \tag{22}$$

At the primary ($u = 1$) and secondary ($u = u_{\pm}$) resonant frequencies, the normalized input admittance is real and equals:

$$g_{in}(u = 1) = r_0 + \frac{\left(g_{L,1} + r_1\right) {k_{01}}^2}{a_1 - b_1 + c_1} + \frac{\left(g_{L,2} + r_2\right) {k_{02}}^2}{a_2 - b_2 + c_2}, \tag{23}$$

$$g_{in}(u = u_{\pm}) = g_L + \frac{bd}{c} + \frac{1}{2}\left(\frac{e}{a - k_{01}^2 - k_{02}^2} - \frac{d}{c}\right) h_{\pm}, \tag{24}$$

with $h_{\pm}$ equal to:

$$h_{\pm} = \left(b \pm \sqrt{b^2 - 4c\left(a - k_{01}^2 - k_{02}^2\right)}\right). \tag{25}$$

D. Power gain

The power gain G_P, commonly referred to as the efficiency of a WPT network, is defined as the quotient of the load power P_L, which is the sum of the power of each individual load, by the input power P_{in}. For the given WPT system, the input power is given by:

$$P_{in} = \frac{1}{2} G_{in} |V_0|^2 \tag{26}$$

and the total power dissipated at the loads is given by:

$$P_L = \frac{1}{2}\left(G_{L,1}|V_1|^2 + G_{L,2}|V_2|^2\right), \tag{27}$$

which gives us the following expression for the power gain:

$$G_P = \frac{P_L}{P_{in}} = \frac{G_{L,1}}{G_{in}}\left|\frac{V_1}{V_0}\right|^2 + \frac{G_{L,2}}{G_{in}}\left|\frac{V_2}{V_0}\right|^2. \tag{28}$$

We can express $\frac{V_1}{V_0}$ and $\frac{V_2}{V_0}$ as a function of the system admittances as:

$$\frac{V_1}{V_0} = \frac{-y_{10}}{y_{11} + Y_{L,1}} \tag{29}$$

and

$$\frac{V_2}{V_0} = \frac{-y_{20}}{y_{22} + Y_{L,2}}, \tag{30}$$

with $Y_{L,i}$ the load admittances.

Using a simple but elaborate algebraic restatement, we can express (28) as:

$$G_P = \frac{k_{01}^2 u^4}{a_1 u^4 - b_1 u^2 + c_1}\frac{g_{L,1}}{g_{in}} + \frac{k_{02}^2 u^4}{a_2 u^4 - b_2 u^2 + c_2}\frac{g_{L,2}}{g_{in}}. \tag{31}$$

For a system with identical receivers (31) simplifies to:

$$G_P = \frac{\left(k_{01}^2 + k_{02}^2\right) u^4}{au^4 - bu^2 + c}\frac{g_L}{g_{in}}, \tag{32}$$

with $g_L = g_{L,1} + g_{L,2}$. At the primary and secondary resonant frequencies, the power gain is given by:

$$G_P(u = 1) = \frac{\left(k_{01}^2 + k_{02}^2\right) g_L}{g_L r_0 \left(g_L + 2r_1\right) + \left(k_{01}^2 + k_{02}^2\right)\left(g_L + r_1\right)} \tag{33}$$

$$G_P(u = u_{\pm}) = \frac{g_L}{g_L + \frac{bd}{c} + \frac{1}{2}\left(\frac{e}{a - k_{01}^2 - k_{02}^2} - \frac{d}{c}\right) h_{\pm}}. \tag{34}$$

E. Transducer gain

The transducer gain G_T is defined as the quotient of the load power P_L by the maximum input power, also called the available power of the generator, P_{AG}. For a fixed P_{AG}, maximizing G_T corresponds to maximizing the amount of power transfer to the load.

For the given CPT system, the available power of the generator is given by:

$$P_{AG} = \frac{|I_s|^2}{8G_S}, \tag{35}$$

which gives us the following expression for the transducer gain:

$$G_T = \frac{P_L}{P_{AG}} = \frac{4G_S(G_{L,1}|V_1|^2 + G_{L,2}|V_2|^2)}{|I_S|^2} \tag{36}$$

We can express V_1 and V_2 as a function of the system admittances and peak value of the current source I_S by multiplying (29) and (30) with V_0, where V_0 is given by:

$$V_0 = \frac{I_S}{Y_S + y_{00} - \frac{y_{01}y_{10}}{y_{11} + Y_{L,1}} - \frac{y_{02}y_{20}}{y_{22} + Y_{L,2}}}. \tag{37}$$

Using a simple but elaborate algebraic restatement, we can express (36) as:

$$G_T = \frac{4 g_{in} g_S}{\left|y_{in} + g_S\right|^2} G_p. \tag{38}$$

At the primary and secondary resonant frequencies, the transducer gain is given by:

$$G_T(u = 1) = \frac{4\left(k_{01}^2 + k_{02}^2\right) g_S g_L^2 \left(g_L + 2r_1\right)}{\left[\left(k_{01}^2 + k_{02}^2\right)\left(g_L + r_1\right) + g_L\left(g_S + r_0\right)\left(g_L + 2r_1\right)\right]^2} \tag{39}$$

TABLE I
NORMALIZED QUANTITIES FOR THE CPT SYSTEM WITH IDENTICAL AND NON-IDENTICAL RECEIVERS.

Normalized quantity	Identical receivers	Non-identical receivers
g_S	0.010	0.010
r_{L_0}	0.010	0.010
r_{L_1}	0.015	0.015
r_{L_2}	0.015	0.025
r_{C_0}	0.005	0.005
r_{C_1}	0.005	0.005
r_{C_2}	0.005	0.010
$g_{L,1}$	0.150	0.150
$g_{L,2}$	0.150	0.200

$$G_T(u=u_\pm) = \frac{4 g_S g_L}{\left[g_S + g_L + \frac{bd}{c} + \frac{1}{2}\left(\frac{e}{a-k_{01}^2-k_{02}^2} - \frac{d}{c}\right) h_\pm\right]^2}. \tag{40}$$

It is worth mentioning that in the lossless case ($r_{C_i} = 0$, $r_{L_i} = 0$), the denominators of (34) and (40) reduce to g_L and $(g_S + g_L)^2$ respectively. Hence, at the secondary resonances, the gains are independent of k_{xy}. Furthermore, when the inductor and capacitor quality factors are high, the coupling-dependent terms in (34) and (40) are typically small with respect to g_L and, therefore, nearly coupling-independent gains can be achieved.

III. RESULTS AND DISCUSSION

A. Identical receivers

We consider a CPT system with identical receivers, using the normalized quantities as given in Tab. I.

Figs. 2 and 3 show G_P and G_T as a function of the coupling coefficient k_{02} for a coupling coefficient k_{01} equal to zero and 0.1 respectively. Note that k_{01} equal to zero corresponds with a system that only has one receiver (i.e., a SISO system), and the result matches the result in [6].

In both scenarios, at the primary resonance, the power gain G_P increases with an increasing coupling coefficient k_{02}. For k_{01} equal to zero, the transducer gain increases with an increasing coupling coefficient for a coupling below the optimal coupling coefficient k_c (i.e., the coupling coefficient that maximizes the power transfer), and decreases again for a coupling coefficient larger than the optimal coupling coefficient. As k_{01} equal to 0.1 is already larger than k_c, the maximum G_T is found at k_{01} equal to zero and decreases with increasing k_{02}.

For a coupling coefficient higher than the bifurcation coupling k_B (i.e. the coupling coefficient that satisfies $k_{02} = \sqrt{a - \frac{b^2}{c} - k_{01}^2}$), the secondary resonances are shown. In both scenarios, both G_P and G_T increase slightly for u_+ and decrease slightly for u_-. We can see clearly the advantage of the coupling-independent modes for CPT applications; we obtain a higher power transfer to the load at the expense of only a small reduction in efficiency.

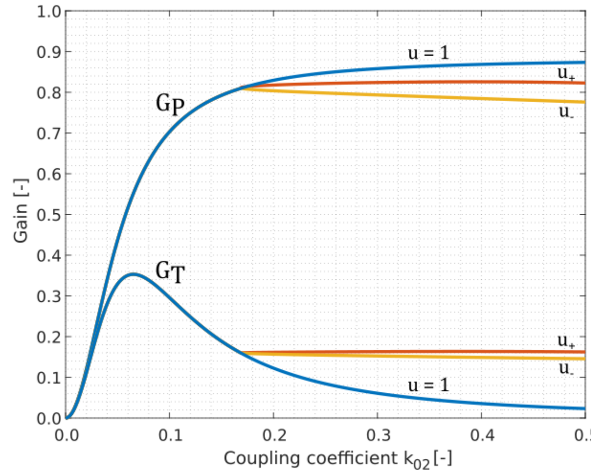

Fig. 2. Power G_P and transducer gain G_T, as a function of the coupling coefficient k_{02} for a coupling coefficient k_{01} equal to zero.

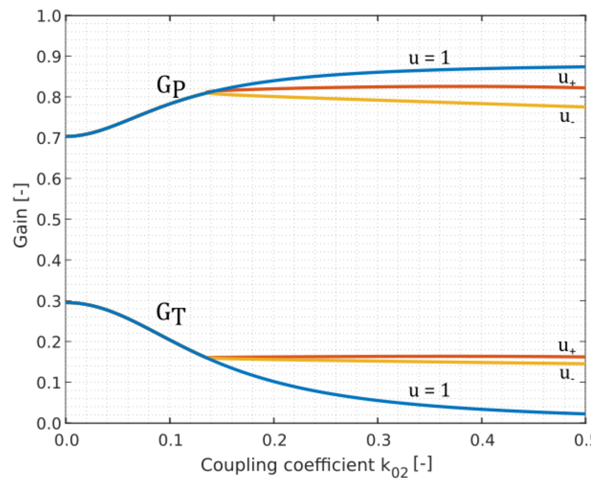

Fig. 3. Power and transducer gain G_P, G_T as a function of the coupling coefficient k_{02} for a coupling coefficient k_{01} equal to 0.1.

Fig. 4 shows the power gain G_P as a function of the coupling coefficients k_{01} and k_{02} for the given system with equal receivers. At the primary resonance, G_P increases for higher coupling coefficients. Note that the side along the k_{02} axis corresponds to Fig. 2. In Fig. 5, G_P is shown as a function of the coupling coefficients k_{01} and k_{02} using u_+ for the sum of the coupling coefficients higher than the bifurcation coupling. A nearly constant G_P can be observed for varying coupling.

Fig. 6 shows G_T as a function of the coupling coefficients k_{01} and k_{02} for the given system with equal receivers. G_T increases with the coupling coefficients until the sum of the coupling coefficients reaches the optimal coupling coefficient. For coupling coefficients $k_{01} + k_{02} > k_c$, the transducer gain G_T decreases for the primary resonance. For

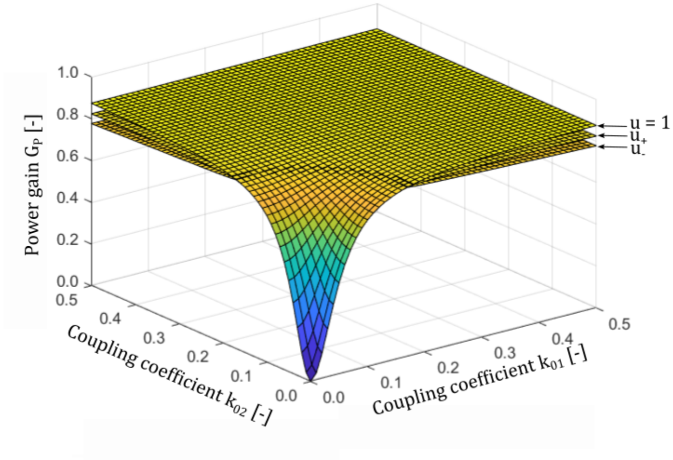

Fig. 4. Power gain G_P as a function of the coupling coefficients k_{01} and k_{02} for a system with two equal receivers.

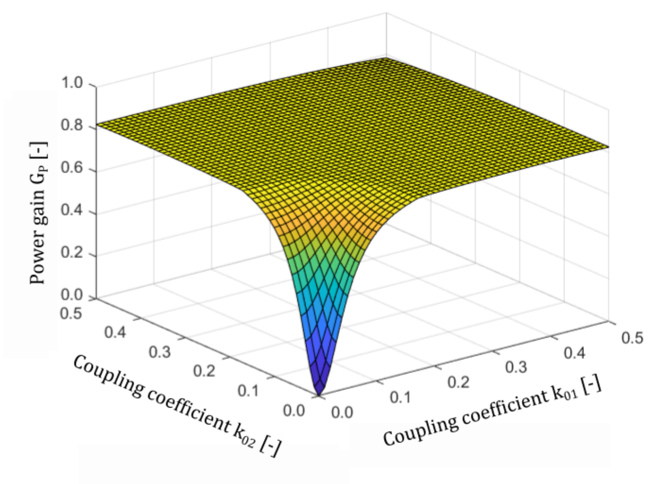

Fig. 5. Power gain G_P as a function of the coupling coefficients k_{01} and k_{02} for a system with two equal receivers using u_+.

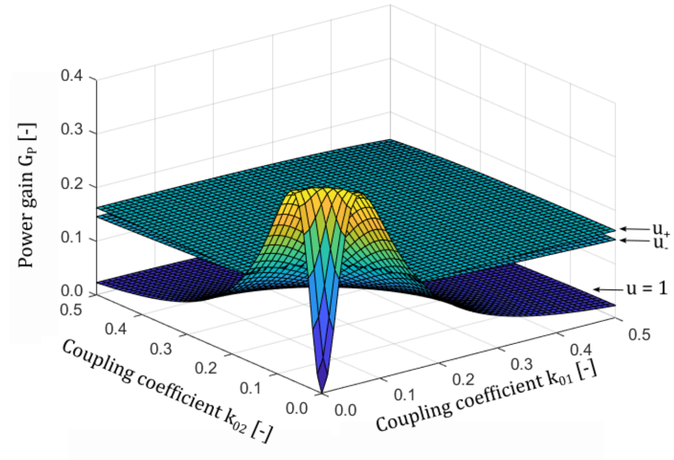

Fig. 6. Transducer gain G_T as a function of the coupling coefficients k_{01} and k_{02} for a system with two equal receivers.

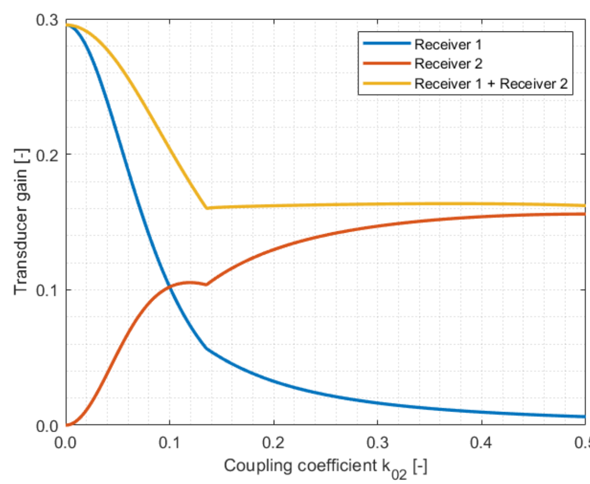

Fig. 7. Transducer gain G_T of the individual (identical) receivers for the system with a coupling coefficient k_{01} equal to 0.1 as a function of the coupling coefficient k_{02}.

coupling coefficients above the bifurcation coupling, when $k_{01} + k_{02} > k_B$, we observe a nearly constant G_T for variable coupling. Note that this nearly constant G_T at the secondary resonances is significantly larger compared to the G_T at the main resonance.

Although we observe a nearly constant power regime when looking at the total system, this is different when looking at each receiver's transducer gain. In Fig. 7, G_T is shown for the individual receivers for a coupling k_{01} equal to 0.1. It is evident that for the individual receivers, the power is still a function of the coupling coefficient.

B. Non-identical receivers

Although practical CPT applications may often aim to use identical receivers, (small) deviations between the receivers can occur. Therefore, it is interesting to look into a system with non-identical receivers as well and we will consider a CPT system with non-identical receivers, using the normalized quantities as given in Tab. I.

Figs. 8 and 9 show G_P and G_T as functions of the coupling coefficient k_{01} and k_{02}. Note that due to the effect of the non-identical receivers, where receiver 1 has higher quality factors for its components compared to receiver 2, the plane is no longer symmetric for both the coupling coefficient axes. Because of this, the variation in G_T is larger compared to the scenario with equal receivers. However, we can still clearly see the higher power transfer benefit of the coupling-independent modes.

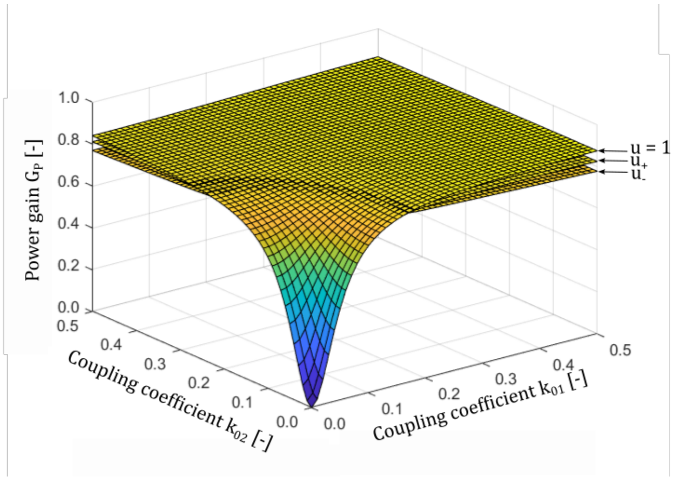

Fig. 8. Transducer gain G_T as a function of the coupling coefficients k_{01} and k_{02} for a system with two unequal receivers.

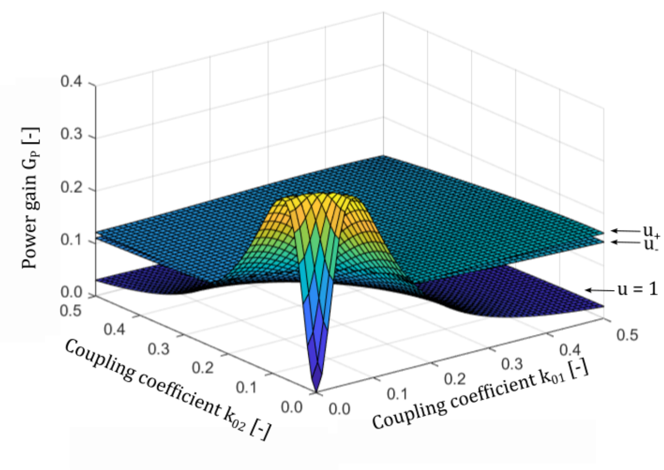

Fig. 9. Transducer gain G_T as a function of the coupling coefficients k_{01} and k_{02} for a system with two unequal receivers.

Conclusion

We analytically determined expressions for the power and transducer gain and showed that these are nearly coupling independent for the secondary resonances, resulting in a nearly constant efficiency and power output for varying coupling.

The results are shown using two illustrative numerical examples, the first using identical, and the second using non-identical receivers, both scenarios clearly show the advantage in terms of power transfer to the load for coupling independent modes.

By accepting a slight decrease in efficiency, it allows for designs that can achieve a higher power transfer to the load for fluctuating coupling.

Circuit simulations and measurements on an implemented SIMO CPT setup to confirm our analytical results are part of future research.

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